Exercise 68

Find the exact value of each expression.

(a) $\arcsin(\sin(5\pi/4))$ (b) $\cos\left(2\sin^{-1}\left(\frac{5}{13}\right)\right)$

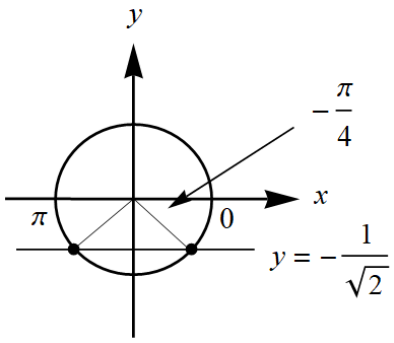
Solution

Part (a)

Since the inverse sine gives a value between $-\pi/2$ and $\pi/2$, $\arcsin(\sin(5\pi/4))$ is not simply $5\pi/4$.

$$\arcsin(\sin(5\pi/4)) = \sin^{-1}\left(\sin\frac{5\pi}{4}\right) = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

We want to find the counterclockwise angle from the positive x-axis to the points on the unit circle that have a vertical distance of $-1/\sqrt{2}$.



The point of interest is in the fourth quadrant because it's between $-\pi/2$ and $\pi/2$. Therefore,

$$\arcsin(\sin(5\pi/4)) = -\frac{\pi}{4}.$$

Part (b)

The inverse sine is an angle. Let θ represent it.

$$\theta = \sin^{-1}\left(\frac{5}{13}\right)$$

Therefore,

$$\begin{aligned}
\cos\left(2\sin^{-1}\left(\frac{5}{13}\right)\right) &= \cos 2\theta \\
&= 1 - 2\sin^2\theta \\
&= 1 - 2(\sin\theta)^2 \\
&= 1 - 2\left(\frac{5}{13}\right)^2 \\
&= \frac{119}{169} \\
&\approx 0.704.
\end{aligned}$$